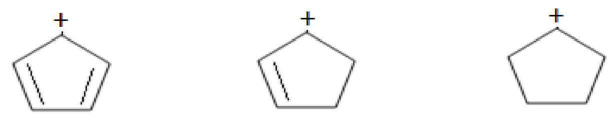
Part Test 1

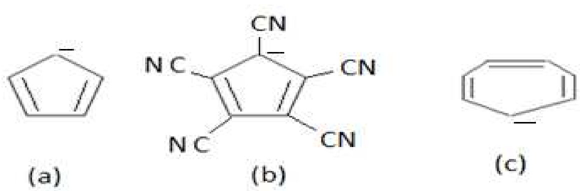
Solutions:

1. (C) Explanation: Basicity α Lone pair availability (Lewis acid-base definition).
   In (c), Nitrogen has pure lone pair while in other cases lone pair is delocalized through resonance. More the delocalization less is the lone pair availability.
2. (B)Explanation: SN1 reactivity α Stability of carbocation formed.

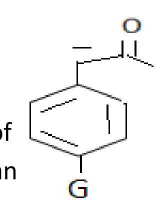


(a) Antiaromatic (b) Aromatic (C) Non-aromatic

   Stability order Aromatic > non-aromatic > antiaromatic
3. (D)Explanation: Compound have more benzenoid structure (having benzene type of structure) are more stable then compound not having benzenoid structure.
   (a) has 2 benzenoid structure (since pi bond in the middle is available for both the rings.)
   (b) and (c) have 1 benzenoid structure.
4. (C)Explanation: (c) is antiaromatic.
5. (A)Explanation: Heat of hydrogenation α (1/stability of alkene). Stability order (a)>(b)>(C) because of more α-hydrogen hence stabilizes via hyperconjugation.
   [Note: Heat of hydrogenation cannot be compared if the compounds do not give the same product on hydrogenation. In the present case all compounds will give same product on hydrogenation.]
6. (A)Explanation: More the stability of conjugated base more is the acidity of corresponding acid.
   (b) is the most stable because of stabilization of –ve charge by resonance as well as –I effect og CN. (c) is antiaromatic and hence least stable.



7. (A)
   Explanation :ity α –ve Charge density.
   [Note: In case of protic solvent like water nucleophilicity order would have been reversed because of salvation of –ve ions. F^- : maximum solvated so least nucleophilic.]
8. (C) Explanation:
   $Co_3F_6^{3-} \rightarrow 3\,Co^+ + 6F^-$
   Co + is hard acid and F- is a hard base. According to HSAB (Hard-soft acid-base) theory hard-hard or soft – soft combinations are stable. I^- is soft base and Ag^+ is weak acid.
9. (B)Explanation: For halogens -I effect predominates over +R effect.
10. (C) Explanation: Intermediate formed during base catalyzed enolization:
    More the –I effect of G more would be the keto-enol ratio.



11. (B) Explanation: Draw the R-S configuration of the compounds .
12. (B) Explanation: (a) and (c) have plane of symmetry while (d) has center of
13. (A) Explanation: achiral means not chiral. (a) has two same group hence can
14. (B) Explanation: (i) and (iii) have two same groups in the carbocation formed as an intermediate.
    (iv) gives distereomers.
15. (D) Example: Ehtylene glycol is one of the example for which gauche form is more than staggard because of intra-moleculate H-bonding between O-H groups.

16. (A)
17. (D) t-BuO$^-$ K$^+$ is a strong bulky base which gives E2 product when reacted with 1°Halide.
18. (B)

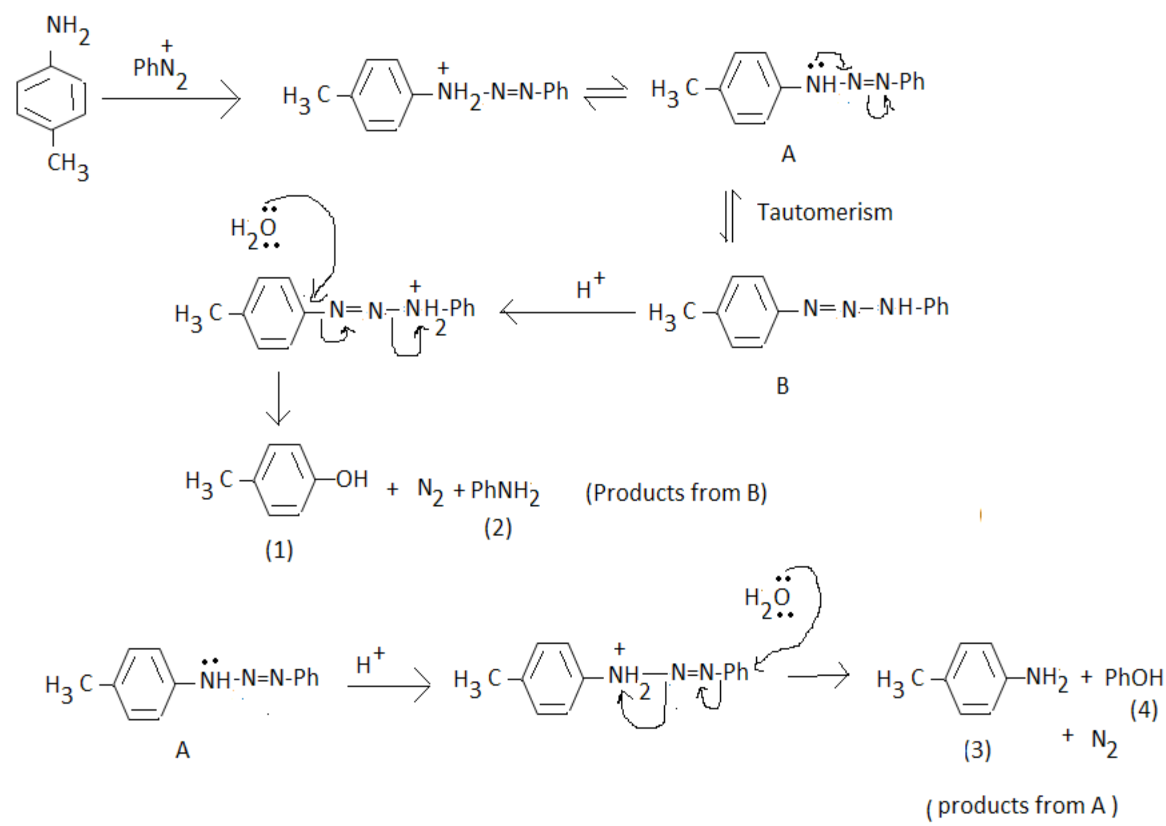
    Yield of (a)/(b) = (No. of 1°H)/No. of 3°H)x (reactivity of 1°H/reactivity of 3°H)

    $= (9/1) \times (1/5) = 9/5 = \frac{(9/14)}{5/14} = \frac{0.64}{0.36} = \frac{64\%}{36\%}$
19. (B) Explanation: A is cis-2-butene and addition of water in presence of alkaline KMnO4 is syn addition of OH-OH, hence product will be meso.
20. (A) Alkyne gives trans-alkene in presence of Li/NH3.

Solutions of subjective problems:

A. Hint: Product obtained by anti elimination of HBr is trans, hence dipole moment =zero

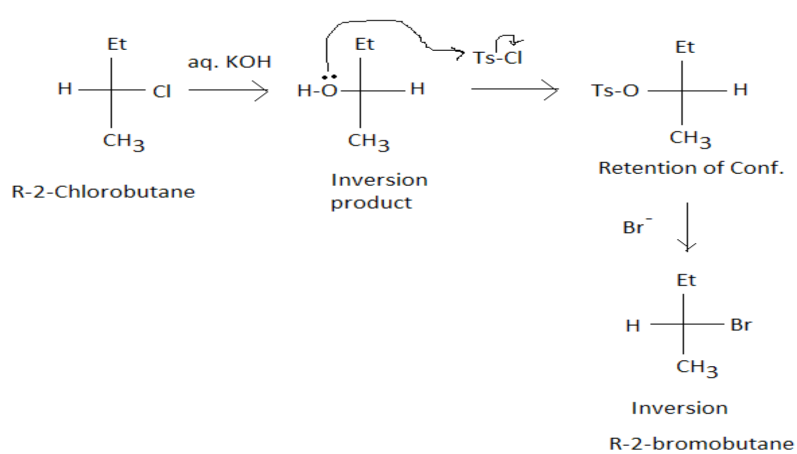
B.



Four products formed are named as (1), (2), (3) & (4).

C. R-2-chlorobutane to R-2-bromobutane.

Two times inversion will give the desired product.



26. $\mid 2\tan 3x \mid 5 = (\tan 3x \mid 1)^2 \mid 4$

Thus the maximum value is e^{-4}

27. If $\lim_{x\to 0, y\to 0}\left(\frac{x}{y}\right) = l_1$, $\lim_{y\to 0}(\lim_{x\to 0}\left(\frac{x}{y}\right)) = l_2$ and $\lim_{x\to 0}(\lim_{y\to 0}\left(\frac{x}{y}\right)) = l_3$ then

l_1 is not defined. l_2 is equal to zero. l_3 is also not defined. Also, two undefined are not equal. Hence none of these.

28. $\lim_{x\to 2}\left[\frac{x^2-1}{x-1} - \frac{2x^2+2x}{x^2-4}\right] = \lim_{x\to 2}\left[(x+1) - \frac{2x^2+2x}{x^2-4}\right]$

a) 1 b) 5/4 c) 3/4 d) none of these

29. $\frac{1}{x-10} + \frac{1}{x-9} + \frac{1}{x-8} + \frac{1}{x-7} + \cdots + \frac{1}{x} + \cdots + \frac{1}{x+8} + \frac{1}{x+9} + \frac{1}{x+10}$ is a

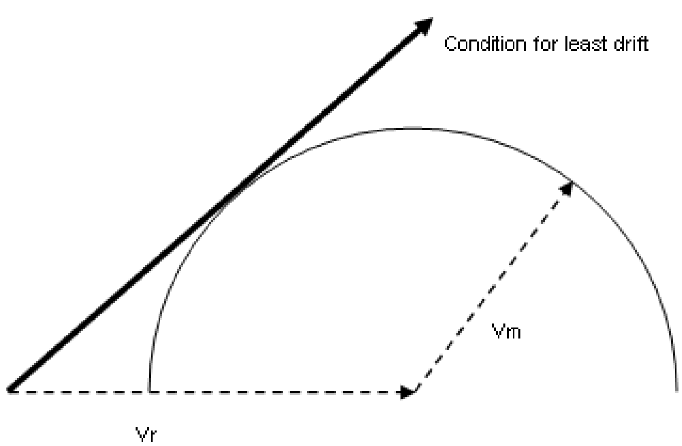
Answer is none of these. That is because the function is not defined at any point at -10, -9,…., 9, 10

30. $\lim_{n\to\infty}\left[\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}\cos\frac{x}{16}\ldots\cos\frac{x}{2^n}\right]$

Multiply and divide by $\sin\frac{x}{2^n}$, you will get $\frac{\sin x}{2^n} \times \frac{1}{\sin\frac{x}{2^n}}$

This tends to $\frac{\sin x}{x}$ as n tends to infinity

31.



Using the above diagram solve it ☺

32. $\lim_{x\to\infty}\sqrt[4]{(x+p)(x+q)(x+r)(x+s)} - x$ is equal to

$\lim_{x\to\infty} x\{\sqrt[4]{(1+p/x)(1+q/x)(1+r/x)(1+s/x)} - 1\}$

Now use the expansion of {1+ax+..}$^{1/4}$ as 1+1/4 ax + …. Where a = p+q+r+s above.

Thus we get answer as $\frac{p+q+r+s}{4}$

33. $\lim_{x\to 0}\frac{sin(1/x)}{sin(1/x)}$ Does not exist because there are many points very close to zero where the function is not defined. Don't confuse it with the case $\lim_{x\to 0}\frac{1/x}{1/x}$. In this case, the limit will be equal to 1

34. Find dy/dx for $\frac{[\sin x]}{x}$ where $[.]$ denotes the greatest integer function.

    The function is not continuous at the points where sin x = 0 or 1. Sin x = -1 will not cause any issues when we deal with continuity. Hence the solution will be c

35. $xy = k(y + \sqrt{y^2 - x^2})$

    None of these is correct because there are slight changes/ mistakes in the options

36. $y = 1 + \frac{a}{x-a} + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$,

    $$1 + \frac{a}{x-a} + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)} = \frac{x}{x-a} + \frac{bx}{(x-a)(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$$
    $$= \frac{x}{x-a}\left\{1 + \frac{b}{(x-b)}\right\} + \frac{cx^2}{(x-a)(x-b)(x-c)} = \frac{x}{x-a}\frac{x}{(x-b)} + \frac{cx^2}{(x-a)(x-b)(x-c)}$$
    $$= \frac{x}{x-a}\frac{x}{(x-b)}\frac{x}{(x-c)}$$

    Take log on both sides and differentiate.. You will get the answer….

37. From a frustum whose two faces have radii a & b and has a length of l, a cylindrical rod has to be cut out. What is the maximum volume of the rod?

38. $\lim_{x\to\infty} x^2\left(2 - \sqrt[5]{1 + \frac{1}{x} + \frac{1}{x^5}} - \sqrt[5]{1 - \frac{1}{x} + \frac{1}{x^5}}\right)$

39. Write the derivative of $(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1} + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $x = \frac{\pi}{4}$

40. [x] for a=1 and b=2.

Physics

41 to 44

First and foremost, find the electric field in between the plates when there is no dielectric: It is given by using guass's law and is equal to $\frac{2-1}{2\epsilon_0 A} = \frac{1}{2\epsilon_0 A}$

Electric field in the region due to dielectric of const 4 is given by $\frac{1}{2\epsilon_0 A} \times \frac{1}{4}$

Electric field in the region due to dielectric of const 2 is given by $\frac{1}{2\epsilon_0 A} \times \frac{1}{2}$

Potential difference is calculated by taking the integral. It will be given by $\frac{d}{2}\left\{\frac{1}{2\epsilon_0 A} \times \frac{1}{4} + \frac{1}{2\epsilon_0 A} \times \frac{1}{2}\right\}$

100r/8

0 current flows

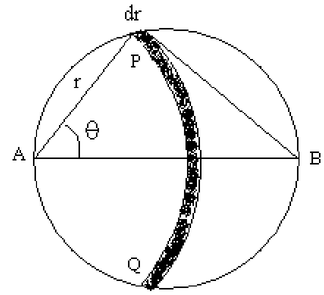
i/8

force is $|\frac{kQq}{r^2}|$

Write the equation ir+q/c=0. Then integrate!

45. CV is the charge on the capacitor C at infinitely long time. C/2V is the charge on the capacitor C/2 at long time. Both will be chaged with the same polarity. Hence the total charge will be 3/2CV.

46. Find the potential at a point on the edge of a thin disc of radius R carrying a uniformly distributed charge with surface density σ.



Let AB be a diameter and A be the point where the potential is to be calculated. From A as centre, we draw tho arcs of radii r and r+dr.

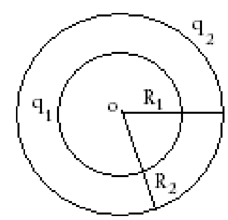
Area of the infinitesimal element region between these two arcs

dA = 2θ x rdr Potential at A due to this element PQ $dV = \dfrac{ds \times \sigma}{4\pi\epsilon_0 r} = \dfrac{2\theta\, rdr\, \sigma}{4\pi\epsilon_0 r} = \dfrac{2\theta\, \sigma dr}{4\pi\epsilon_0}$

From ΔAPB, r = 2R cosθ dr = -2R sinθ dθ

Again, $dV = \dfrac{-4\sigma\theta\, R \sin\theta\, d\theta}{4\pi\epsilon_0}$ $V = \displaystyle\int_{\pi/2}^{0} \frac{-\sigma\theta\, R \sin\theta\, d\theta}{\pi\epsilon_0} = \frac{\sigma R}{\pi\epsilon_0}$

47. Two conectric spheres of radii R_1 and R_2 ($R_1 < R_2$) have charges q_1 and q_2 respectively distributed uniformly over their surfaces. Find the potential difference between the two surfaces.



Potential of inner sphere = potential due to q_2 + potential due to q_1

$= \dfrac{q_2}{4\pi\epsilon_0 R_2} + \dfrac{q_1}{4\pi\epsilon_0 R_1}$

Potential of outer sphere = potential due to q_2 + potential due to q_1 $= \dfrac{q_2}{4\pi\epsilon_0 R_2} + \dfrac{q_1}{4\pi\epsilon_0 R_2}$

Potential Difference $= \dfrac{q_1}{4\pi\epsilon_0}\left(\dfrac{1}{R_2} - \dfrac{1}{R_1}\right)$

48. Initial charges of plate 1 of C_1, $q=C_1V=+48\mu C$. So on plate 2 it is -48μC.

Initial charges of plate 3 of C_2, $q=C_2V=+72\mu C$. So on plate 4 it is -72μC.

Now in the new connection, suppose charge q is flown for C_2 to C_1. So we have charges on plates as

1:48-q, 2:-48+q, 3:72-q, 4:-72+q

Measuring potential in the circuit, we have $V_1+V_2=0$ or (48-q)/4 + (72-q)/6=0 or q=57.6 μC.

So the charges on the plates are

1: -9.6 μC, 2:9.6 μC, 3:14.4 μC, 4:-14.4 μC

49. (d) $K(q_a/a + q/b + q/c) = 0$ (potential at shell A is zero) $q_a = -qa(1/b + 1/c)$

Charge flown to earth = q + qa(1/b +1/c)

50. (a) $K(q_a/a + q/b + q_c/c) = 0$ (potential at shell A is zero)

and, $K(q_a + q + q_c)/c = 0$ (potential at shell C is zero)

Solve the two equation to get $q_c = -q(1/a - a/b)/(1/a - 1/c)$ Q flow to earth q + q(1/a -1/b)/(1/a - 1/c)

51. (c) $K(q_a/a + q/b + q_c/c) = 0$ (potential at shell A is zero)

and, $K(q_a + q + q_c)/c = 0$ (potential at shell C is zero)

Solve the two equation to get $q_a = -q(1/b - 1/c)/(1/a - 1/c)$

Hence charge that will flow from shell A to earth = - qa(1/b +1/c) + q(1/b - 1/c)/(1/a - 1/c)

52. (a)Since potential at the surface of Shell A is zero, the potential at any point inside it will be zero as no charge is inside the shell A.
53. (b)potential at all the shells will be zero.

    Hence, $q_a/a + q_b/b + q_c/c = 0$ $q_a/b + q_b/b + q_c/c = 0$ $q_a/c + q_b/c + q_c/c = 0$

    For positive value of a,b and c q_a, q_b and q_c must be zero.

    Hence charge flown will q.