

Remainders

A sample of the unique Teacher-Student dialogue format material that we provide

Student: Sir, what is the remainder when 2^{2000} is divided by 17

Teacher: See in this problem u need to get something of the form $(x+y)^n$ and hope to get one of x or y as 1 or -1 or some very small integer!

In most such problems u will find something of this kind!

Do u see 17 as something of this kind ?

Student: Yes... May be I don't know

Teacher: Arrey! is it $1+2^4$?

Student: yeah . 17

Teacher: 2^{2000} this has to be written in terms of 17 how do u do that?

Student: Take $2^4 \dots 2^{2000} = (2^4)^{500}$ right?

Teacher: now it is only 1 step from the final answer! can u think?

Student: I add and subtract 1 $(2^4+1-1)^{500}$

Teacher: perfect.. so it becomes?

Student: $(17-1)^{500}$

Teacher: cool.. so how many terms of this expansion will not be divisible by 17 and what will that term be?

Student: don't know

Teacher: see it is $(x-1)^n$ form... right? And you want the remainder when you divide this by x. Do u see the similarity?

so what are the terms of the expansion of $(x-1)^n$ which will not be divisible by x

Student: no idea!

Teacher: ok so whenever we have x^r , it will be divisible by x except when $r=0$?

Student: 0

Teacher: great. So what do we need to find in the above problem so that we get the term of the expansion which is not divisible by x?

Student: so we need 2 find d rth term and $r=0$

Teacher: yes perfect; so what is the rth term in $(x-1)^n$ where $r = 0$?

Student: ${}^{500}C_0(17)^{500}(-1)^0$

Teacher: Think again! I mean coefficient of x^0 in $(x-1)^n$

Student: ${}^nC_r \cdot a^{n-r} \cdot b^r$ where $a=x$, $b=-1$

Teacher: yes. $r=0$?

Student: 0

Teacher: u want coefficient of x^0 right? so when x^0 then a^{n-r} . So $0=n-r$ so $r=n$

Student: Am repeating it again. First we need 2 find d thing which we will divide in the numerator, so then we do sum "lafdas" Then it will not be divisible or we get d remainder r only when the rth coefficient is 0. So we find that

Teacher: yes ok.. so u need to find the coefficient of x^0

Student: yeah so is q ka answer we r getting 1

Teacher: yes!

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Student: yeah that's rite :D

Teacher: Now a tough problem

Student: yeah armed

Teacher: so Now, solve this question remainder when 45^{1000} is divided by 4

Student: $(7^7-4)^{1000}$

Teacher: OMG.. what is all this !!

Student: ☺... arrey we bring 4 na

Teacher: yes

Student: they will cancel

Teacher: so 4 has to come as a multiple of something

Student: why $4n+1$?

Teacher: $45=4.11+1$

Student: y not $3n+2$

Teacher: because we need to find the remainder when dividing by 4

Student: oo aacha thanks I'll solve

Teacher: In the last question we needed to find the remainder when dividing by 17. So we tried to bring 17

Student: $(44+1)^{1000}$

Teacher: yes

Student: answer is 1

Teacher: yes

Student: yeah got it

Teacher: now do this for 45^{1000} divided by 7
it might take 2-3 steps but fear not!

Student: for this it would be $49-4$

Teacher: yes'

Student: then we divide by 4

Teacher: no

Student: and we take that out

Teacher: no not yet! :(

Student: aacha baba

Teacher: see first step was brilliant. u can take 2 things $49-4$ or $42+3$
which ever u want so we have $(49-4)^{1000}$

what will be the coefficient of 49^0 in this expansion?

see sometimes that divided by y thing u will have to leave :)

but as I said .. all this will come with some practice

Teacher: so it will be 4^{1000} which can be written as 16^{500} ?

Student: y u want to write that again! That's d answer na?

Teacher: because how do we reduce 4^{1000} ?

Student: but that is the answer

Teacher: hey remainder is always less than 7 na?

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Student: :D

Teacher: if u divide something by 7 can the remainder be more than 7

Teacher: So what will u do now? Reduce it to on of 2 things. either take 2^{2000} or 16^{500}

Student: yeah $(14+2)^{500}$

Teacher: yes good work so we again get to 2^{500} ? right? That is why this is a tougher problem :P 2^3 is 8 or what?

Student: $(8-1)^{500/3}$

Teacher: but $500/3$ is not an integer

Student: $(7+1)^{500/3}$

Teacher: I think u need one small change here but $500/3$ is not an integer dear!

Student: change we need

Teacher: so what we can do is write $2^{500}=2^{498}.4$

Student: What is this! $498.4 \sim 500$

Teacher: $2^{498} 2^2$

Student: ooooo ok

Teacher: $2^{(498+2)}$

Teacher: now 2^{498} divide this by 7 and multiply the remainder by 2^2 .. ok?

Student: yeah

Teacher: this is a slightly tough/different(step) so what will it be?

Student: what... we can write 2 in terms of 8

Teacher: yes

Student: $(7+1)^{498/3}$

Teacher: yes and a multiply by 4 at the end of it right?

Student: $(7+1)^{166}$

Teacher: yeah great.. so it is?

Student: yeah done thanks

Teacher: Say the answer a)1 b)2 c)4 d) none of these!

Student: 4

Teacher: yes gr8 :)

Student: thank u sirji

Teacher: You want another one before we say good bye to this type of questions?

Student: yeah sure

Teacher: 9^{153} divided by 13..

Student: it is 1 and I guess that is right?

Teacher: gr8. How?

Student: $(26+1)^{51}$

Teacher: Wrong ☹

Student: why?

Teacher: 9^{153}

Student: Arrey you told it was right

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Teacher: it is 3^{306} . Your answer is right but method is slightly wrong.

Student: 9^{153}

Teacher: ok tell me again

Student: 27^{51}

Teacher: no

Student: $(26+1)^{51}$

Teacher: it is $(9^3)^{51}$

Student: oppppppsy yeah got it

Teacher: Yes so try it again and reach the solution :). 1 is the right answer btw

Student: ma *tukkas* r rite

Teacher: that is good :) It helps in the exams ;)

Student: so its $(3^2)^{153}$; so u can rite it as $(3^3)^{101}$; 3^3 is $26+1$

Teacher: chi chi mistake again.. small one but wrong is wrong!

Student: yeah 102

Teacher: hmm...

Student: $(3^3)^{102}$

Student: $(26+1)^{102}$ so answer is 1

Teacher: yes good. I hope you can solve questions of this type now :)

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