Where Vectors fits:
'Vectors' is an essential part of physics. It will be present in mechanics, electrostatics, gravitation, optics and what not! Hence is something which should be very well understood by every student. The present content is designed to take care of the physics part only, not Mathematics.

Prerequisites:

- Basic Trigonometry (Definition of sin, cos, tan)
- Pythagoras theorem.

Topics to be studied:

- What is a vector
- Addition of vectors
- Dot product and cross product

Motivation:
Have you ever measured the time to go to your school? 0.25 hrs/ 15 mins/900 s. Tried to measure the distance to your school? 1 km. Just by saying that your school is 1 km from your house, can your friend locate your school precisely? No. You have to tell him which direction to take. You have to tell him to go in the north direction. This is where vectors come into picture.
The quantity where direction does not matter is called a scalar. A quantity where the direction makes huge sense is vector. So the distance of your school from your home is a scalar. But the Location of your school from your home is a vector.

Representation:
A vector is represented by an arrow, originating at one point and ending at another. Its length gives the "magnitude" of the vector, while the orientation gives the direction of that vector.

These are two vectors which have the same direction but one is twice the other!	These are two negative vectors	These are two equal Vectors

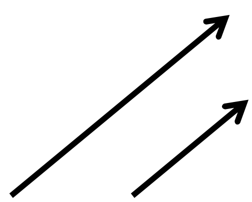
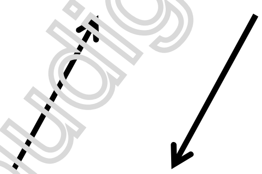
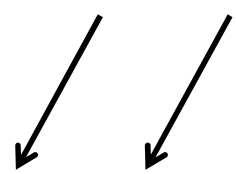

		

Image: 1 a	Image: 1 b	Image: 1 c

Equality of Vectors:
Two vectors are said to be equal if they have the same magnitude are in the same direction. (Image 1c) They need not originate from the same point!

Negative Vectors:
Two vectors are negative if they are parallel, equal in magnitude but in opposite direction (Image 1 b)

Zero Vector:
The magnitude of such a vector is zero. Its direction is indeterminate. A practical example is that if you are at your school, you have to move 0 distance to reach your school and you can go in any direction. ☺

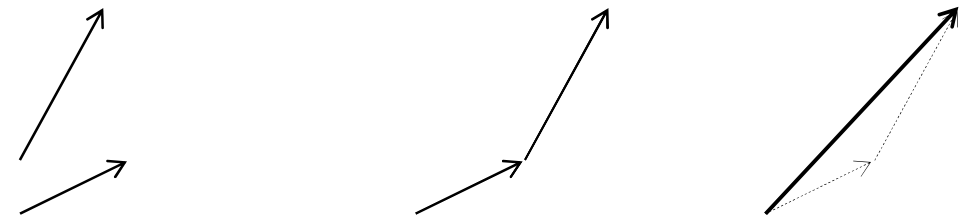
Unit Vector:
Any vector whose length is 1 unit. Standard unit vectors along x, y, z directions are given by i, j, k.

Vector Addition:
Addition of two vectors is not as direct as addition of scalars. When we add scalars, we hardly care. But vector addition is something more different. There are two ways to add a vector.

a) Triangle Law b) Parallelogram Law

Triangle Law:
If two vectors $\bar{a}$ and b are to be added, we join them so that the head of one is at the tail of the other. The line joining the tail of the first and the head of the second will give the resultant vector.

Triangle Law of Addition



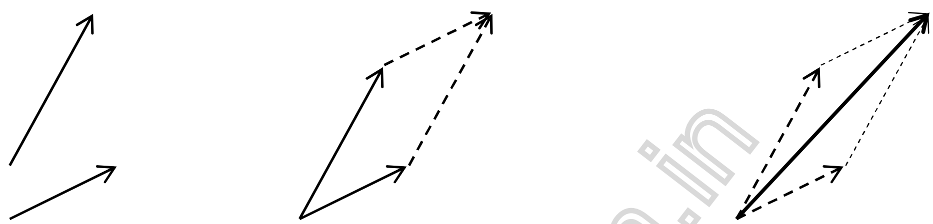
The first diagram shows three vectors. In the second we join the head of one to the tail. In the third, we depict the resultant vector.

Image: 2

Parallelogram Law:

If two vectors ā and b are to be added, we join them so that the tail touches each other. Then using these two sides, we complete the parallelogram. The line joining the meeting point of the tail to the other vertex gives the resultant vector.

Parallelogram Law of Addition



The first diagram shows three vectors. In the second we join the tail of both and complete the parallogram. In the third, we depict the resultant vector.

Image: 3

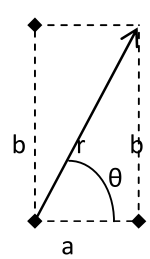
This proof below needs your understanding of distance from basic coordinate geometry module, just the distance formula.

Vector Notation:

Vector can be written in two ways, one by the lengths in each of the direction for instance 100 m west and 100 m north or equivalently as √200 m in the north-west.

In the **Cartesian representation**, a vector is denoted as ai + bj on a 2-Dimensional plane and as *ai + bj + ck* on a 3-dimensional plane. i, j, k denote unit vector along positive x, y, z directions respectively.

In the **polar representation**, it is denoted equivalently as (r, θ) where r is the length of the vector.



Polar form to Cartesian form

In the diagram alongside, see that the length of the vector is r.

So the base $a = r \cos \theta$ and $b = r \sin \theta$

Thus $(r, \theta) = r \cos \theta\ i + r \sin \theta\ j$

(r gives the magnitude of the vector)

Cartesian form to Polar form

If a and b are known, $r = \sqrt{a^2+b^2}$ due to Pythagoras theorem.

and $\tan \theta = b/a$, thus $\theta = \tan^{-1}(b/a)$

Thus, $a + ib = (\sqrt{a^2+b^2}, \tan^{-1}(b/a))$

Example 1: Find the magnitude of the vector 12i+3j+4k

Soln: $r^2 = 12^2+3^2+4^2 = 169$. Thus, r=13.

Example 2: if a=(10, 53°) write it in Cartesian form (given : sin 53°=3/5)

Soln: 10 cos 53 i + 10 sin 53 j = 8i+6j

Example 3: if a=(10, 53°), b=(5, 37°) Express a+b in both polar and Cartesian form

Soln: a+b=(10 cos 53+5 sin 37)i + (10 sin 53+5 cos 37)j = 11i + 10j. Now you can finish the second part

Example 4: if a=(15, 53°), b = 5i+4j. Express a-b in both polar and Cartesian forms

Soln: a= 12i+9j, a-b=7i+5j. Second part is left for you.

Now with the notation we have above, we will try to have a relook at vector addition and see if we can extract more quantitative information

Vector Addition Revisited:

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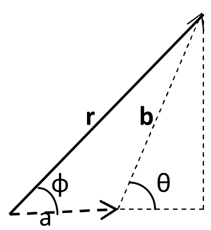
<td>
</td>
<td>Assumption: a, b are two vectors of length a & b respectively and make an angle theta with each other.
$r \cos \phi = a + b \cos \theta$
$r \sin \phi = b \sin \theta$
Square and add the above two, $r^2=a^2+b^2\cos^2\theta + 2ab \cos \theta + b^2\sin^2\theta$
$r^2=a^2+b^2 + 2ab \cos \theta$. Thus we have the value of r.

As a very simple exercise you are left to prove that
$\phi = \tan^{-1}\{b \sin\theta/(a+b \cos\theta)\}$</td>
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The above illustration is to show the addition of two vectors which are in polar form. Now lets try to see the sum of two vectors in Cartesian form.

v_1 = ai+bj and v_2 = ci+dj

then, the sum v_1+v_2 = (a+c)i+(b+d)j (This is called the component-wise addition of vectors)

Example 1: Find the sum of the vectors 3i+5j+3k and -3i+2j+3k

Soln: Sum is taken component wise. Hence it is 7j+6k

Example 2: The resultant of the vectors 4i+xj+3k and 2yi+2j+xk is parallel to the vector j+k. Find x, y

Soln: Sum is taken component wise. Hence it is 7j+6k

Example 3: if |a|+|b|=|a+b| then what can you say about a and b?

Soln: Think carefully. It means that the sum of two sides of a triangle is equal to the third side. Hence it means that they are parallel vectors. This explanation is intentionally vague so that you apply some thought.

Example 4: if |a|-|b|=|a+b| then what can you say about a and b?

Soln: Think carefully. It means that the difference of two sides of a triangle is equal to the third side. Hence, it means that the sum of two sides is equal to the third side. Here the answer will be that they are in opposite direction to each other.

Scalar product or Dot product:

The scalar product of two vectors a and b are represented as a.b = |a||b| cos θ where |a| and |b| are the magnitudes of the vectors and θ is the angle between the vectors.

If $a = a_xi + a_yj + a_zk$ and $b=b_xi + b_yj + b_zk$ then $a.b = a_x b_x+a_y b_y+a_z b_z$

Vector Product or Cross Product:

It is represented as a x b = |a||b| sin θ n, where n is the unit vector in the direction given by right hand screw rule going from vector a to b.

Note: a x b = - b x a

When we have $a \times b = (a_xi + a_yj + a_zk) \times (b_xi + b_yj + b_zk)$

Note: Dot product is a scalar quantity while cross product gives another vector

Example 1: If the vectors 2i+4j+5k and xi+yj+10k are parallel find x,y

Soln: They are parallel, hence their cross product is zero. Hence

Example 2: The resultant of vectors i+2j and 3i+4k are perpendicular to the vector xi+3j. Find x.

End of theory

Example 1: If a, b, c are three vectors, such that a+b=2c, show that O is the midpoint of AB

Solution: We have to prove that vector BC = CA

Now, OB+BC=OC (Triangle Law) (ie b + BC = c)

OC+CA=OA (Triangle Law) (ie c + CA = a)

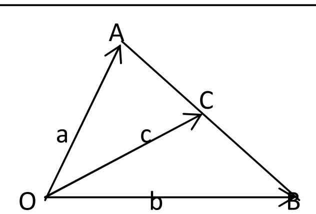
Adding the above two, b+a+BC=c+c+CA

Thus, BC = CA (using a+b=2c)

Hence we have proved the equality of vectors BC and CA.

They are equal only if their magnitudes are equal.

Thus C is the mid point of AB.



For example 11